Economics of deforestation

Coefficient interpration

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Linear regressions: typical models

Level–level regression

$$y = \alpha + \beta x + \varepsilon \tag{1}$$

Log–level regression

$$ln(y) = \alpha + \beta x + \varepsilon \tag{2}$$

Level–log regression

$$y = \alpha + \beta ln(x) + \varepsilon \tag{3}$$

Log–log regression

$$ln(y) = \alpha + \beta ln(x) + \varepsilon \tag{4}$$

Regression table

Dependent variable	wage	ln wage	wage	ln wage
	Level-level	Log-level	Level-log	Log-log
	(1)	(2)	(3)	(4)
Education	922.00***	0.290***		
	(210.32)	(0.045)		
ln Education			3622.50***	3.00***
			(839.55)	(0.450)
Observations controls	5000	5000	5000	5000
Adj. R^2	0.02	0.04	0.03	0.05

Note: These are completely made up values

Interpretation

Level–level regression

$$y = \alpha + \beta x + \varepsilon \tag{5}$$

$$\frac{\delta y}{\delta x} = \beta \tag{6}$$

$$\delta y = \delta x \cdot \beta \tag{7}$$

$$\Delta y = \Delta x \cdot \beta \tag{8}$$

A 1 unit increase in x leads to a β unit change in y

1 year of additional education increases wages on average by 922 dollar per month

What if y is a dummy?

Interpretation

Log–level regression

$$\ln y = \alpha + \beta x + \varepsilon \tag{9}$$

$$(\frac{\delta}{\delta x}) \ln y = \alpha + \beta x + \varepsilon \tag{10}$$

$$\frac{dy}{y} = dx \cdot \beta \tag{11}$$

$$\frac{dy}{y} \cdot 100 = dx \cdot 100\beta \tag{12}$$

$$\%\Delta y = dx \cdot 100\beta \tag{13}$$

more precise would be: $$\%\Delta y = dx \cdot 100(e^{\beta} - 1) \tag{14}$$

A 1 unit increase in x leads to a $100 \cdot \beta$ percentage change in y

1 year of additional education increases wages on average by ca. 29%

What if x is a dummy?

Interpretation

Level–log regression

$$y = \alpha + \beta \ln x + \varepsilon \tag{15}$$

$$(\frac{\delta}{\delta x})y = \alpha + \beta \ln x + \varepsilon \tag{16}$$

$$\mathrm{d}y = \beta \frac{\mathrm{d}x}{x} \tag{17}$$

$$100 \cdot \mathrm{d}y = \beta \cdot 100 \frac{\mathrm{d}x}{x} \tag{18}$$

$$100 \cdot \mathrm{d}y = \beta \cdot \%\Delta x \tag{19}$$

$$\mathrm{d}y = \frac{\beta}{100} \cdot \%\Delta x \tag{20}$$

A 1 percent increase in x leads to a $\frac{\beta}{100}$ change in y

A 1 percent increase in years of education increases wages on average by 36.23 dollar per month.

Interpretation

Log–log regression

$$\ln y = \alpha + \beta \ln x + \varepsilon \tag{21}$$

$$(\frac{\delta}{\delta x}) \ln y = \alpha + \beta \ln x + \varepsilon \tag{22}$$

$$\frac{\mathrm{d}y}{y} = \beta \cdot \frac{\mathrm{d}x}{x} \tag{23}$$

$$100 \cdot \frac{\mathrm{d}y}{y} = \beta \cdot 100 \cdot \frac{\mathrm{d}x}{x} \tag{24}$$

$$\%\Delta y = \%\Delta x \tag{25}$$

A 1 percent increase in x leads to a β percent change in y

A 1 percent increase in the years of education increases wages on average by 3 percent.